CHAPTER 4 Study Guide and Review

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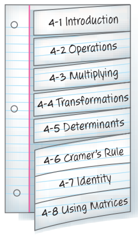
FOLDABLES Study Organizer — GET READY to Study

Be sure the following Key Concepts are noted in your Foldable.



Key Concepts

Matrices (Lesson 4-1)

- A matrix is a rectangular array of variables or constants in horizontal rows and vertical columns.
- Equal matrices have the same dimensions and corresponding elements are equal.

Operations (Lessons 4-2, 4-3)

- Matrices can be added or subtracted if they have the same dimensions. Add or subtract corresponding elements.
- To multiply a matrix by a scalar k, multiply each element in the matrix by k.
- Two matrices can be multiplied if and only if the number of columns in the first matrix is equal to the number of rows in the second matrix.
- Use matrix addition and a translation matrix to find the coordinates of a translated figure.
- Use scalar multiplication to perform dilations.

Transformations (Lesson 4-4)

- To rotate a figure counterclockwise about the origin, multiply the vertex matrix on the left by a rotation matrix.

Identity and Inverse Matrices (Lesson 4-7)

- An identity matrix is a square matrix with ones on the diagonal and zeros in the other positions.
- Two matrices are inverses of each other if their product is the identity matrix.

Matrix Equations (Lesson 4-8)

- To solve a matrix equation, find the inverse of the coefficient matrix. Then multiply each side of the equation by the inverse matrix.

Key Vocabulary

Cramer's Rule (p. 201)
determinant (p. 194)
dilation (p. 187)
dimension (p. 163)
element (p. 163)
equal matrices (p. 164)
identity matrix (p. 208)
inverse (p. 209)
matrix (p. 162)
matrix equation (p. 216)
reflection (p. 188)
rotation (p. 188)
scalar multiplication (p. 171)
translation (p. 185)

Vocabulary Check

Choose the correct term from the list above to complete each sentence.

1. The matrix $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ is a(n) ______ for multiplication.

2. ______ is the process of multiplying a matrix by a constant.

3. A(n) ______ is when a figure is moved around a center point.

4. The ______ of $\begin{bmatrix} -1 & 2 \\ 2 & -3 \end{bmatrix}$ is -1.

5. A(n)______ is the product of the coefficient matrix and the variable matrix equal to the constant matrix.

6. The ______ of a matrix tell how many rows and columns are in the matrix.

7. A(n) ______ is a rectangular array of constants or variables.

8. Each value in a matrix is called an ______.

9. If the product of two matrices is the identity matrix, they are ______.

10. ______ can be used to solve a system of equations.

11. (A)n ______ is when a geometric figure is enlarged or reduced.

12. A(n) ______ occurs when a figure is slid from one location to another on the coordinate plane.

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Lesson-by-Lesson Review

4-1 Introduction to Matrices (pp. 162–167)

Solve each equation.

13. $\begin{bmatrix} 2y - x \\ x \end{bmatrix} = \begin{bmatrix} 3 \\ 4y - 1 \end{bmatrix}$

14. $\begin{bmatrix} 7x \\ x + y \end{bmatrix} = \begin{bmatrix} 5 + 2y \\ 11 \end{bmatrix}$

15. $\begin{bmatrix} 3x + y \\ x - 3y \end{bmatrix} = \begin{bmatrix} -3 \\ -1 \end{bmatrix}$

16. $\begin{bmatrix} 2x - y \\ 6x - y \end{bmatrix} = \begin{bmatrix} 2 \\ 22 \end{bmatrix}$

17. FAMILY Three sisters, Tionna, Diana, and Caroline each have 3 children. Tionna's children are 17, 20, and 23 years old. Diana's children are 12, 19, and 22 years old. Caroline's children are 6, 7, and 11 years old. Write a matrix of the children's ages. Which element represents the youngest child?

Example 1 Solve $\begin{bmatrix} 2x \\ y \end{bmatrix} = \begin{bmatrix} 32 + 6y \\ 7 - x \end{bmatrix}$.

Write two linear equations.

$2x = 32 + 6y$

$y = 7 - x$

Solve the system of equations.

$2x = 32 + 6y$	First equation
$2x = 32 + 6(7 - x)$	Substitute $7 - x$ for y.
$2x = 32 + 42 - 6x$	Distributive Property
$8x = 74$	Add $6x$ to each side.
$x = 9.25$	Divide each side by 8.

To find the value of y, substitute 9.25 for x in either equation.

$y = 7 - x$	Second equation
$= 7 - 9.25$	Substitute 9.25 for x.
$= -2.25$	Simplify.

The solution is (9.25, –2.25).

4-2 Operations with Matrices (pp. 169–176)

Perform the indicated matrix operations. If the matrix does not exist, write *impossible*.

18. $\begin{bmatrix} -4 & 3 \\ -5 & 2 \end{bmatrix} + \begin{bmatrix} 1 & -3 \\ 3 & -8 \end{bmatrix}$

19. $[0.2 \quad 1.3 \quad -0.4] - [2 \quad 1.7 \quad 2.6]$

20. $\begin{bmatrix} 1 & -5 \\ -2 & 3 \end{bmatrix} + \frac{3}{4}\begin{bmatrix} 0 & 4 \\ -16 & 8 \end{bmatrix}$

21. $\begin{bmatrix} 1 & 0 & -3 \\ 4 & -5 & 2 \end{bmatrix} - 2\begin{bmatrix} -2 & 3 & 5 \\ -3 & -1 & 2 \end{bmatrix}$

22. $\begin{bmatrix} 90 & 70 & 85 \\ 72 & 53 & 97 \\ 84 & 61 & 79 \end{bmatrix} - \begin{bmatrix} 93 & 77 & 91 \\ 83 & 52 & 92 \\ 83 & 64 & 89 \end{bmatrix}$

Example 2 Find $A - B$ if $A = \begin{bmatrix} 3 & 8 \\ -5 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} -4 & 6 \\ 1 & 9 \end{bmatrix}$.

$A - B = \begin{bmatrix} 3 & 8 \\ -5 & 2 \end{bmatrix} - \begin{bmatrix} -4 & 6 \\ 1 & 9 \end{bmatrix}$ Matrix subtraction

$= \begin{bmatrix} 3 - (-4) & 8 - 6 \\ -5 - 1 & 2 - 9 \end{bmatrix}$ Subtract.

$= \begin{bmatrix} 7 & 2 \\ -6 & -7 \end{bmatrix}$ Simplify.

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4-3 Multiplying Matrices (pp. 177–184)

Find each product, if possible.

23. $\begin{bmatrix} 2 & 7 \end{bmatrix} \cdot \begin{bmatrix} 5 \\ -4 \end{bmatrix}$

24. $\begin{bmatrix} 8 & -3 \\ 6 & 1 \end{bmatrix} \cdot \begin{bmatrix} 2 & -3 \\ 1 & -5 \end{bmatrix}$

25. $\begin{bmatrix} 3 & 4 \\ 1 & 0 \\ 2 & 5 \end{bmatrix} \cdot \begin{bmatrix} -2 & 4 & 5 \\ 3 & 0 & -1 \\ 1 & 0 & -1 \end{bmatrix}$

26. SHOPPING Mark went shopping and bought two shirts, three pairs of pants, one belt, and two pairs of shoes. The following matrix shows the prices for each item respectively.

[\$20.15 \$32 \$15 \$25.99]

Use matrix multiplication to find the total amount of money Mark spent while shopping.

Example 3 **Find XY if $X = \begin{bmatrix} 6 & 4 \end{bmatrix}$ and $Y = \begin{bmatrix} 2 & 5 \\ -3 & 0 \end{bmatrix}$.**

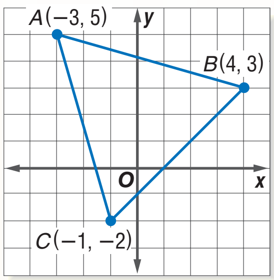
$XY = \begin{bmatrix} 6 & 4 \end{bmatrix} \cdot \begin{bmatrix} 2 & 5 \\ -3 & 0 \end{bmatrix}$ Write an equation.

$= \begin{bmatrix} 6(2) + 4(-3) & 6(5) + 4(0) \end{bmatrix}$ Multiply columns by rows.

$= \begin{bmatrix} 0 & 30 \end{bmatrix}$ Simplify.

4-4 Transformations with Matrices (pp. 185–192)

For Exercises 27–30, use the figure to find the coordinates of the image after each transformation.



27. translation 4 units right and 5 units down

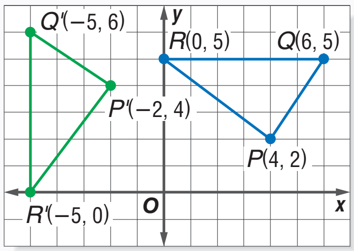
28. dilation by a scale factor of 2

29. reflection over the y-axis

30. rotation of 180°

31. MAPS Kala is drawing a map of her neighborhood. Her house is represented by quadrilateral $ABCD$ with $A(2, 2)$, $B(6, 2)$, $C(6, 6)$, and $D(2, 6)$. Kala wants to use the same coordinates to make a map one half the size. What will the new coordinates of her house be?

Example 4 **Find the coordinates of the vertices of the image of $\triangle PQR$ with $P(4, 2)$, $Q(6, 5)$, and $R(0, 5)$ after a rotation of 90° counterclockwise about the origin.**



Write the ordered pairs in a vertex matrix. Then multiply by the rotation matrix.

$$\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} 4 & 6 & 0 \\ 2 & 5 & 5 \end{bmatrix} = \begin{bmatrix} -2 & -5 & -5 \\ 4 & 6 & 0 \end{bmatrix}$$

The coordinates of the vertices of $\triangle P'Q'R'$ are $P'(-2, 4)$, $Q'(-5, 6)$, and $R'(-5, 0)$.

Mixed Problem Solving
For mixed problem-solving practice, see page 929.

4-5 Determinants (pp. 194–200)

Find the value of each determinant.

32. $\begin{vmatrix} 4 & 11 \\ -7 & 8 \end{vmatrix}$

33. $\begin{vmatrix} 6 & -7 \\ 5 & 3 \end{vmatrix}$

34. $\begin{vmatrix} 12 & 8 \\ 9 & 6 \end{vmatrix}$

35. $\begin{vmatrix} 2 & -3 & 1 \\ 0 & 7 & 8 \\ 2 & 1 & 3 \end{vmatrix}$

36. $\begin{vmatrix} 7 & -4 & 5 \\ 1 & 3 & -6 \\ 5 & -1 & -2 \end{vmatrix}$

37. $\begin{vmatrix} 6 & 3 & -2 \\ -4 & 2 & 5 \\ -3 & -1 & 0 \end{vmatrix}$

38. GEOMETRY Alex wants to find the area of a triangle. He draws the triangle on a coordinate plane and finds that it has vertices at (2, 1), (3, 4) and (1, 4). Find the area of the triangle.

Example 5 **Evaluate $\begin{vmatrix} 3 & 6 \\ -4 & 2 \end{vmatrix}$.**

$$\begin{vmatrix} 3 & 6 \\ -4 & 2 \end{vmatrix} = 3(2) - (-4)(6) \quad \text{Definition of determinant}$$

$$= 6 - (-24) \text{ or } 30 \quad \text{Simplify.}$$

Example 6 **Evaluate $\begin{vmatrix} 3 & 1 & 5 \\ 1 & -2 & 1 \\ 0 & -1 & 2 \end{vmatrix}$ using expansion by minors.**

$$\begin{vmatrix} 3 & 1 & 5 \\ 1 & -2 & 1 \\ 0 & -1 & 2 \end{vmatrix} = 3\begin{vmatrix} -2 & 1 \\ -1 & 2 \end{vmatrix} - 1\begin{vmatrix} 1 & 1 \\ 0 & 2 \end{vmatrix} + 5\begin{vmatrix} 1 & -2 \\ 0 & -1 \end{vmatrix}$$

$$= 3(-4 - (-1)) - 1(2 - 0) + 5(-1 - 0)$$

$$= -9 - 2 - 5 \text{ or } -16$$

4-6 Cramer's Rule (pp. 201–207)

Use Cramer's Rule to solve each system of equations.

39. $9a - b = 1$
$3a + 2b = 12$

40. $x + 5y = 14$
$-2x + 6y = 4$

41. $4f + 5g = -2$
$-3f - 7g = 8$

42. $-6m + n = -13$
$11m - 6n = 3$

43. $6x - 7z = 13$
$8y + 2z = 14$
$7x + z = 6$

44. $2a - b - 3c = -20$
$4a + 2b + c = 6$
$2a + b - c = -6$

45. ENTERTAINMENT Selena paid \$25.25 to play three games of miniature golf and two rides on go-karts. Selena paid \$25.75 for four games of miniature golf and one ride on the go-karts. Use Cramer's Rule to find out how much each activity costs.

Example 7 **Use Cramer's Rule to solve $5a - 3b = 7$ and $3a + 9b = -3$.**

$$a = \frac{\begin{vmatrix} 7 & -3 \\ -3 & 9 \end{vmatrix}}{\begin{vmatrix} 5 & -3 \\ 3 & 9 \end{vmatrix}} \qquad \text{Cramer's Rule} \qquad b = \frac{\begin{vmatrix} 5 & 7 \\ 3 & -3 \end{vmatrix}}{\begin{vmatrix} 5 & -3 \\ 3 & 9 \end{vmatrix}}$$

$$= \frac{63 - 9}{45 + 9} \qquad \text{Evaluate each determinant.} \qquad = \frac{-15 - 21}{45 + 9}$$

$$= \frac{54}{54} \text{ or } 1 \qquad \text{Simplify.} \qquad = \frac{-36}{54} \text{ or } -\frac{2}{3}$$

The solution is $\left(1, -\frac{2}{3}\right)$.

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4-7 Identity and Inverse Matrices (pp. 208–215)

Find the inverse of each matrix, if it exists.

46. $\begin{bmatrix} 3 & 2 \\ 4 & -2 \end{bmatrix}$

47. $\begin{bmatrix} 8 & 6 \\ 9 & 7 \end{bmatrix}$

48. $\begin{bmatrix} 0 & 2 \\ 5 & -4 \end{bmatrix}$

49. $\begin{bmatrix} 6 & -1 & 0 \\ 5 & 8 & -2 \end{bmatrix}$

50. CRYPTOGRAPHY Martin wrote a coded message to his friend using a coding matrix, $C = \begin{bmatrix} 3 & 1 \\ 2 & 4 \end{bmatrix}$. What is Martin's message if the matrix he gave his friend was $\begin{bmatrix} 26 & 12 \\ 80 & 80 \\ 75 & 25 \\ 24 & 38 \\ 94 & 98 \\ 32 & 24 \\ 53 & 101 \end{bmatrix}$? (*Hint*: Assume that the letters are labeled 1–26 with A = 1 and _ = 0.)

Example 8 Find the inverse of $S = \begin{bmatrix} 3 & -4 \\ 2 & 1 \end{bmatrix}$.

First evaluate the determinant.

$$\begin{vmatrix} 3 & -4 \\ 2 & 1 \end{vmatrix} = 3 - (-8) \text{ or } 11$$

Then use the formula for the inverse matrix.

$$S^{-1} = \frac{1}{11}\begin{bmatrix} 1 & 4 \\ -2 & 3 \end{bmatrix}$$

4-8 Using Matrices to Solve Systems of Equations (pp. 216–222)

Solve each matrix equation or system of equations by using inverse matrices.

51. $\begin{bmatrix} 5 & -2 \\ 1 & 3 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 16 \\ 10 \end{bmatrix}$

52. $\begin{bmatrix} 4 & 1 \\ 3 & -2 \end{bmatrix} \cdot \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 9 \\ 4 \end{bmatrix}$

53. $3x + 8 = -y$
$4x - 2y = -14$

54. $3x - 5y = -13$
$4x + 3y = 2$

55. SHOES Joan is preparing a dye solution for her shoes. For the right color she needs 1500 milliliters of a 63% concentration solution. The store has only 75% and 50% concentration solutions. How many milliliters of 50% dye solution should be mixed with 75% dye solution to make the necessary amount of 63% dye solution?

Example 9 Solve $\begin{bmatrix} 4 & 8 \\ 2 & -3 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 12 \\ 13 \end{bmatrix}$.

Step 1 Find the inverse of the coefficient matrix.

$$A^{-1} = \frac{1}{-12 - 16}\begin{bmatrix} -3 & -8 \\ -2 & 4 \end{bmatrix} \text{ or } -\frac{1}{28}\begin{bmatrix} -3 & -8 \\ -2 & 4 \end{bmatrix}$$

Step 2 Multiply each side by the inverse matrix.

$$-\frac{1}{28}\begin{bmatrix} -3 & -8 \\ -2 & 4 \end{bmatrix} \cdot \begin{bmatrix} 4 & 8 \\ 2 & -3 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix}$$
$$= -\frac{1}{28}\begin{bmatrix} -3 & -8 \\ -2 & 4 \end{bmatrix} \cdot \begin{bmatrix} 12 \\ 13 \end{bmatrix}$$
$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = -\frac{1}{28}\begin{bmatrix} -140 \\ 28 \end{bmatrix}$$
$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ -1 \end{bmatrix}$$